Exam 2 Math 213 March 11, 2026

- No calculators, books, or notes are allowed. Show all your work clearly.

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1. A particle moves in space with an acceleration given by: $\mathbf{a}(t) = <0, \sin t,\ \cos t>$. The particle has an initial velocity $\mathbf{v}(0) = <1,\ -1,\ 0>$ and an initial position $\mathbf{r}(0) = <0,\ 0,\ 2>$

a) (6 pts) Find the velocity vector function $\mathbf{v}(t)$

b) (6 pts) Find the position vector function $\mathbf{r}(t)$

2. Consider the curve defined by the vector function $\mathbf{r}(t) = <0, \cos^3 t,\ \ \sin^3 t>$

a) (8 pts) Calculate the length of the path from $t=0$ to $t=\dfrac{\pi}{4}$.

b) (6 pts) Find the unit tangent vector $\mathbf{T}(\mathrm{t})$ and the curvature κ for this curve.

3. (8 pts) Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ If the limit doesn't exist, prove it by the two path test.

4. (10 pts) Let $w=x^2y+z^2$. The variables $x,\ y,$ and z are functions of t given by:

$x(t)=\cos(t)$, $\ y(t)=\sin(t)$, $\ z(t)=t$. Find $\dfrac{dw}{dt}$ at $t=\dfrac{\pi}{3}$ (Use partial derivatives – DO NOT substitute directly before taking derivatives).

5. Consider the surface $z=x^2+y^2-2x-4y$.

a) (6 pts) Find the equation of the tangent plane to the surface at the point (2, -1, 5).

b) (4 pts) Use the differential dz to estimate the value of z at the point (2.01, -0.98).

6. Let $f(x,y)=xe^{xy}$.

a) (6 pts) Find the gradient vector ∇f at the point P(1, 0).

b) (4 pts) Calculate the directional derivative of f at P(1, 0) in the direction of $\mathbf{v}=\langle 3, -4\rangle$.

7. (12 pts) Find all local maximums, local minimums, and saddle points for the function:

$f(x,y)=x^3-3x+3y^2-6y$ and classify each critical point.

8. (12 pts) Use the method of Lagrange Multipliers to find the maximum and minimum values of the function $f(x,y) = x + 2y$ subject to the constraint $x^2+y^2=5$.

9. (8 pts) Find the second-degree Taylor polynomial $P_2(x,y)$ for the function $f(x,y)=\cos(x)\sin(y)$ centered at the point $(0, \dfrac{\pi}{2})$

10. (8 pts) Evaluate the double integral over the rectangular region $R=[0,1]\times[0,2]$:

$$\iint_R (4x+6y^2)\, dA$$